Jules Verne's
Journey through Interplanetary Space

Bachelor's Thesis

based on the course *Introduction to Geophysics*

Institute for Geophysics, Astrophysics and Meteorology (IGAM)
University of Graz

Daniel Kraft

Supervisor: Dr.rer.nat. Ulrich Foelsche

July 8th, 2010

1 Introduction

In his novel "Hector Servadac" (see [15] and [12]), Jules Verne describes how a group of 36 people is mysteriously transferred onto a comet (called "Gallia") as it comes in contact with the Earth on New Year's night of an unspecified year in the 19th century. Despite the unavoidable violence such a collision must bring with it, they are all unharmed; and as Gallia also fetches a part of the Earth's atmosphere as well as soil, water, vegetation and animals, they can live there and are taken on a two years' tour through the solar system.

Gallia's orbit is elliptical and strongly eccentric, so they first approach the Sun until reaching the perihelion on Jan, 15th, within Venus' orbit. Then Gallia recedes further away for exactly one terrestrial year into the colder regions — its aphelion lies even outside of Jupiter's orbit, so Jules Verne lets Gallia's little population explore regions of space that were completely out of reach to humankind of his time (except for the use of telescopes) and are even today for manned space flight.

To his great pleasure, the peculiar professor Palmyrin Rosette, Gallia's discoverer while still on Earth, takes part in the journey; additionally, some of the other inhabitants are somewhat educated in natural philosophy and astronomy in particular and have books about those topics with them — which means that Jules Verne has good opportunities to make knowledge of his time about the solar system known to his readers. As usual with his works, of course he also makes use of those possibilities.

In this text, I will take a look at certain claims of his concerning some aspects of Gallia itself and its orbit, to compare and verify statements in the book. There are a lot of numerical figures, but as it is mostly a work of fiction instead of science, they are usually given only roughly (additionally at some places fractional values are approximated by simple fractions like 0.133 being "nearly" $\frac{1}{7}$) — so it is clear that we can't reasonably expect very precise agreement between Verne's claims and calculations of mine. However, a not insignificant part of his statements agrees within some 10 % of error tolerance which means that he was probably well aware of the expected correlation between the involved quantities.

1.1 Units

As opposed to the English translation [12], the French original text [15] nearly exclusively uses metric units so that it is straight-forward to transfer figures given to SI for calculations: Lengths are given in metres, masses in kilograms, times in every-day units of seconds, minutes, hours, days, months and years (explicitly with the corresponding terrestrial meaning), dates in the usual terrestrial calendar and temperatures in centigrades. Because the year when the collision happens is not explicitly specified, I will number it as year 1 in the following — it is however known that this year is a leap-year (see [12, ch. I/18, line 4284]).

For large lengths however (especially astronomical distances), also the unit "lieue" (league) is used; I will denote one league as $1\,\mathrm{l}$ (the unit of litres is never used throughout this text). According to [13], a possible interpretation that fits in well with the other units is the *metric* league with a length of exactly 4 km.

According to [15, ch. I/8], the average distance between Earth and Sun is

$38 \cdot 10^6$ l. When this is compared to the value of 1 AU, the length of one league must be 3937 m — within the precision of Verne's rounded numbers, this clearly leads to the conclusion that the choice above is reasonable.

1.2 Historical Considerations

"Hector Servadac" was published in 1877 (see [12, preface]). The third edition of Isaac Newton's "Philosophiae Naturalis Principia Mathematica" [6] was published in 1726 more than a century before, which describes the Newtonian axioms of motion, his theory of gravity and derives Kepler's laws of planetary motion from movement within a central force-field (e.g., pages 13f, 38, 52, 390ff, 395ff, 403, 409). This means that at this time, the classical laws of mechanics and celestial motion were already well known and well analyzed. In fact, Jules Verne explicitly mentions Kepler's laws and Newton's law of gravitation; see for instance [12, ch. I/15, line 3657] and [12, ch. II/7, line 8280].

Later in the 18th century — also significantly before Jules Verne's time — measurements of both the gravitational constant (actually, Earth's mean density which implies it, most famously [2]) and the solar parallax (as described in [5]) allowed to estimate the mean distance between Earth and Sun, and from this both the Sun's mass and the distances of other planets from the Sun by Kepler's laws (those distances are also explicitly mentioned in the novel, for instance at [15, ch. II/9]). Thus it is clear that we can expect that all necessary aspects of celestial mechanics were very well known and readily available to Verne at his time.

For estimation of mean temperature depending on distance from the Sun (as I do in Section 5), things look different however. While Lord Kelvin introduced his "absolute thermometric scale" that fixed the point of absolute zero in 1848 (see [10]) and this could (or could not) have been known to Jules Verne, today's Stefan-Boltzmann law which will play a significant role in the modeling of temperature was only published in 1879 as [9] — so two years too late for Verne to know about it. It seems that here he had no real tool in hand for estimation of the temperature values stated, even if he wanted to be as precise and theoretically correct as possible. Even worse, according to [9], the two Frenchmen Dulong and Petit — whose experimental data Stefan used — had come up with their own empirical formula to describe radiation, namely that it is proportional to 1.0077^T (as compared to Stefan's T^4 that is nowadays accepted). Because they died in 1838 and 1820, respectively, *their* results were possibly known to Verne; so he might even have had a wrong relation available instead of no one at all. Whether he actually made use of this or tried to model the temperature of Gallia at all, is not clear of course, though.

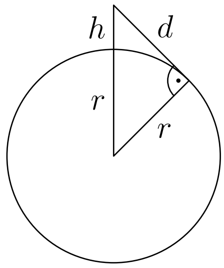

Figure 1: Deriving distance of horizon.

2 Dimensions of Gallia

The first things that Captain Hector Servadac and his companion Ben Zoof, two French soldiers formerly stationed in the French colony of Algeria, notice after they have been transferred to Gallia together with some parts of Algeria they were on, are that their weight is strongly diminished, the globe has assumed "a more decided convexity" [12, ch. I/5, line 749], the Sun is rising in the West and setting in the East now, and one (synodical) day is reduced to half its length, namely 12 hours [12, ch. I/6, line 1085]. That is, Gallia is considerably smaller than the Earth, has a much weaker gravity and rotates with double angular speed but in retrograde fashion. Additionally, their new position is on the equator and Gallia's axis of rotation is not inclined but precisely orthogonal to its plane of orbit (so that there are no seasons in the classical sense), see [12, ch. I/8, line 1574].

2.1 Size and Near Horizon

Later in Verne's description, Servadac and a party of Russians circumnavigate Gallia's equator in a schooner yacht (the "Dobryna"). This leads to the only real statement about Gallia's size, and no attempts are made to calculate it in a more precise way — but probably the sailors are well trained in their craft and are indeed able to estimate the equator's circumference without much error. According to [15, ch. II/5], this circumference is 2300 km. As there is no contrary data, I'll assume that Gallia's shape is spherical, which implies that it's mean radius is then $R = 366\,\text{km}$ or as of [12, ch. II/4, line 7547] about $\frac{1}{16}$ of the Earth's mean radius $R_E = 6371\,\text{km}$; more precisely, it is $R \approx \frac{R_E}{17.41}$.

This directly correlates with Verne's mention of a more convex globe; by diminishing it's diameter, the horizon gets drawn nearer. The situation is sketched in Figure 1. Assume you're on a (perfect) sphere of radius r with your eyes at height h over the ground (where $h \ll r$). Then the "horizon" is visible at a distance of d just where your line of sight is tangent to the sphere. By Pythagoras' theorem, we find $(r+h)^2 = r^2 + d^2$ or

$$d = \sqrt{h^2 + 2hr} = \sqrt{r\left(2h + \frac{h^2}{r}\right)} \approx \sqrt{2hr}. \tag{1}$$

So for a fixed "observation point" (i.e., fixed h), the distance d to the horizon is proportional to $\sqrt{r}$ when changing the globe's radius r. In [15, ch. I/5], it is claimed that for a certain point on the coast, the distance to the new horizon is only 10 km where it had been 40 km before. I would probably not expect myself to be able to estimate such distances more or less accurately "by sight", but I guess a French officer can.

The decrease by a factor of $4 = \sqrt{16}$ fits in very well with the later statement that the radius is reduced to $\frac{1}{16}$ compared to the Earth. So Jules Verne was very likely aware of this geometric relationship. Using Equation 1 with Earth's mean radius of R_E and $d = 40\,\text{km}$, the height of this particular spot is $h = 126\,\text{m}$; then on Gallia, the correct distance using this height would have been $d' = 9.59\,\text{km}$ instead of 10 km, or rather $\frac{1}{\sqrt{17.41}} = \frac{1}{4.17}$ than $\frac{1}{4}$ of 40 km — but this is clearly only a rounding issue.

2.2 Gravity, Mass and Density

In the second part of the novel, driven by Palmyrin Rosette, Gallia's inhabitants measure the mass and density of their new home. This is done via finding the changed value of gravity acceleration, which seems to be a very reasonable way to do it. A test-mass of 1 kg is weighed and the apparent mass displayed (which is less than 1 kg) in relation to the correct 1 kg gives the new acceleration g on Gallia as compared to $g_E = 9.81\,\frac{\text{m}}{\text{s}^2}$, because the attracting force is diminished in that ratio and thus the weighing is influenced.

Of course, scales can not be used for this, because they only find the relative mass of some object compared to known test-masses — and thus would still yield the correct result independently of gravity (as long as it is not completely zero). This is also explained by Jules Verne in [12, ch. II/5, line 7763], so they venture to use a spring-balance instead, because this device actually measures the real force of attraction and is thus susceptible to a change in gravity acceleration.

Fortunately, they are able to get both the spring-balance and a test-mass of exactly 1 kg despite their somewhat isolated position. The apparent mass as displayed by the balance is 133 g (see [15, ch. II/8]), but note that the balance is manipulated and displays 1 kg for actually only 750 g, which is found out later [15, ch. II/16] — with this,

$$g = g_E \cdot \frac{133\,\text{g}}{1\,\text{kg}} \cdot \frac{3}{4} = 0.98\,\frac{\text{m}}{\text{s}^2}.$$

In theory, this value is given as sum of gravitational acceleration (inwards) and acceleration due to centrifugal force (outwards). For mass M of Gallia and rotational period $T = 12\,\text{h}$, it is (positive means inward)

$$g = \frac{GM}{R^2} - \frac{4\pi^2}{T^2}R, \tag{2}$$

where the two terms correspond to gravitation and centrifugal force due to Gallia's rotation, respectively. With already known R, it is trivial to calculate the second expression to $\frac{4\pi^2}{T^2}R = 7.74 \cdot 10^{-3}\,\frac{\text{m}}{\text{s}^2}$ (less than 1 % of the measured total). Now, from Equation 2 we can calculate Gallia's mass according to the measurement of g:

$$M = \frac{R^2}{G}\left(g + \frac{4\pi^2}{T^2}R\right) = 1.98 \cdot 10^{21}\,\text{kg}$$

This finally leads to a mean density of $\rho = \frac{M}{\frac{4}{3}\pi R^3} = 9639\,\frac{\text{kg}}{\text{m}^3}$, which is much larger than that of the Earth (or any other planet).

So far my own analysis; in the book, they proceed a little different. After having determined g with the test-mass, they measure the density of a particular substance, of which all of Gallia's surface seems to be made (unknown on Earth or at least to any of them). A cube of $1\,\text{dm}^3$ of this substance weighs 1.43 kg on the spring-balance according to [15, ch. II/8]. Considering the changes in gravity and the wrong calibration, this results in a corrected mass of 10.752 kg or a surface-density of $\rho_s = 10752\,\frac{\text{kg}}{\text{m}^3}$. Note that in fact this value is independent of the $\frac{3}{4}$ correction they only find out later, so their result is immediately correct.

From this and the assumption that this substance "no doubt constitutes the sole material of the comet, extending from its surface to its innermost depths" [12, ch. II/7, line 8334] they calculate the mass as product of volume and density. Although the surface-density is admittedly already very high and thus there's probably not much margin for a much heavier core in a differentiated body, this still seems to be a very questionable assumption and practice. Especially as it is completely unnecessary to make it at all, because the more direct method via g itself would also have worked.

Anyways, here again the density from Verne's stated values and that from direct estimation of Gallia's mass match up with not much more than 10 % of error. So he clearly hit the correct numbers again.

Interestingly though, due to the indirect method of calculating Gallia's mass where the "correction factor" $\frac{3}{4}$ of the wrongly calibrated balance does not matter, they arrive at the correct result despite not knowing about their wrong measurements (at least the same result as if they had already known and cared about that factor) — so in some sense, they actually did it the right way, although they could not have expected this problem. However, they later only find out about the wrong balance (partly) because Rosette's observations of Nerina (a satellite of Gallia) don't match up with his calculations, where he blames a wrong value for Gallia's mass (see [12, ch. II/14, line 10424]) — but even though the spring-balance displayed wrong weights, their estimated mass was already correct. This is apparently something that Jules Verne messed up because he didn't think about it in depth.

2.3 Valuable Soil

After the calculation of Gallia's density, Rosette comes to the bold proposition that the "strange substance" that makes up Gallia's soil "contains 70 per cent. of tellurium, and 30 per cent. of gold" (see [12, ch. II/7, line 8417] or [15, ch. II/8]) — because "the sum of the specific gravities of these two substances is 10, precisely the number that represents Gallia's density" (if using Verne's rounded numbers, the mean density is "about" $10\,\frac{\text{g}}{\text{cm}^3}$).

According to [1], the densities of those two substances are $\rho_{\text{Au}} = 19.32\,\frac{\text{g}}{\text{cm}^3}$ and $\rho_{\text{Te}} = 6.25\,\frac{\text{g}}{\text{cm}^3}$. It is not clear whether the percentages given are fractions of Gallia's *volume* or *mass* — let's first assume those are volume-parts. Then some volume V of Gallia's soil consists of $0.3V$ gold and $0.7V$ tellurium, so its mass is $m = V \cdot (0.3\rho_{\text{Au}} + 0.7\rho_{\text{Te}})$ which gives a mean density of

$$\rho_{\text{Vol}} = 0.3\rho_{\text{Au}} + 0.7\rho_{\text{Te}} = 10171\,\frac{\text{kg}}{\text{m}^3}.$$

This value fits well into the picture, as it is near the cited "10" and also just between the two densities found above from the gravity and surface substance. Additionally, the nice formula as weighted mean of the two densities just matches Verne's description.

If on the other hand the fractions are parts of the mass, a piece of the substance with mass m contains $0.3m$ gold and $0.7m$ tellurium. The respective volume is $V = m \cdot \left(\frac{0.3}{\rho_{\mathrm{Au}}} + \frac{0.7}{\rho_{\mathrm{Te}}}\right)$ so in this case the overall density is rather the harmonic mean, namely

$$\rho_{\mathrm{Mass}} = \left(\frac{0.3}{\rho_{\mathrm{Au}}} + \frac{0.7}{\rho_{\mathrm{Te}}}\right)^{-1} = 7841\,\frac{\mathrm{kg}}{\mathrm{m}^3}.$$

This value clearly is out of range, so Jules Verne obviously meant percent of volume — as was already suspected above.

It is however also curious that Rosette seems to decide about the substance "just" because of its density, especially with inexact, rounded numbers — I should try selling a mixture of lead and iron with exactly silver's density to my local jeweler, maybe this will work out. But possibly Rosette already knew the substance from other reasons and was only ensured about its nature and exact mixture by the measurement result.

3 Gallian Atmosphere

Gallia does obviously possess an atmosphere that is suited to support life, because otherwise Verne's characters would have been in quite a dire situation. It probably took away a part of the Earth's during the collision (see for instance [12, ch. I/15, line 3436] and [12, ch. II/12, line 9618]), so I will assume that its composition is similar to the Earth's atmosphere with the same amount of oxygen and the same molar mass.

Verne mentions in the book at certain places (first at [12, ch. I/5, line 731]) that breathing has become more difficult and the atmospheric pressure is much less than that of the Earth — but still only to the extent that it forms no real physical problem for anyone on Gallia.

In this section, I'll estimate the surface pressure of Gallia from the statements given in the book, and see if this is a reasonably possible value or not; note that Gallia is much smaller and has a much weaker gravity than the Earth according to Section 2!

3.1 Surface Pressure

While Jules Verne mentions in his novel that Gallia's surface pressure is much lower than the Earth's, no explicit value (or ratio between those two) is given (only the in my opinion useless comment "that the column of air above the Earth's surface had become reduced by one-third of its altitude" in [12, ch. I/7, line 1339]). However, according to [15, ch. I/7] and [12, ch. I/7, line 1327], the boiling temperature of water is reduced from the "usual" 100 °C at one atmospheric pressure on Earth to 66 °C — this corresponds to a significantly reduced pressure, according to [15, ch. I/7] the same as on a mountain 11 km high on Earth.

From those two statements, we can arrive at an explicit value of Gallia's atmospheric pressure. To be precise, those observations are made not at surface-level but on top of a cliff at the former Algerian coast; so maybe some 100 m or 200 m above the sea. However, because that is in any case only an insignificant fraction of the barometric scale height on Gallia (as seen in Section 3.2), it is surely justified to identify the pressure there with the real surface pressure in good approximation.

Water begins to boil when the outer pressure equals the saturation pressure at its temperature, so Gallia's surface pressure is the saturation pressure at 66 °C. The saturation pressure is given by the Clausis-Clapeyron equation (see [8, p. 186]):

$$e_s(T) = e_{s0} \cdot \exp\left(\frac{l_w}{R_w}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right), \tag{3}$$

where e_{s0} is the saturation pressure at temperature T_0 as reference-point, R_w is the specific gas constant for water vapour and l_w is water's evaporation enthalpy. Unfortunately, Equation 3 is only valid approximately in this form because the temperature-dependence of l_w has been neglected; but for a small temperature range it is a good approximation.

[7] is a table of saturation pressure values, which does not give $e_s(66\,^\circ\mathrm{C})$

directly but instead we find

$$\begin{aligned} e_s(64.053\,^\circ\mathrm{C}) &= 240\,\mathrm{hPa} \\ e_s(67.518\,^\circ\mathrm{C}) &= 280\,\mathrm{hPa}. \end{aligned}$$

With the help of Equation 3, we can interpolate between those values. Solving for $\frac{l_w}{R_w}$, we obtain

$$\frac{l_w}{R_w} = \ln\left(\frac{e_s(T)}{e_{s0}}\right) \cdot \left(\frac{1}{T_0} - \frac{1}{T}\right)^{-1},$$

which can now be calculated for the temperature range about 66 °C using the data points from the table. Having this at hand, it is easy to find

$$p_0 = e_s(66\,^\circ\mathrm{C}) = 262\,\mathrm{hPa}$$

as surface pressure of Gallia from Equation 3. I will use this value later on (as opposed to other possibilities explored below).

As for the pressure 11 km above the surface of the Earth (which is the second statement made by Jules Verne), the "classical" barometric formula for an isothermic atmosphere (see [8, p. 164]),

$$p(z) = p_0 \cdot \exp\left(-\frac{z - z_0}{H}\right),$$

with temperature of 0 °C results in $p_1 = 256\,\mathrm{hPa}$, which is quite close to the pressure p_0. However, in reality the atmosphere is not isothermic of course; and the table at [8, p. 166] gives the correct pressure at this height as $p_2 = 226\,\mathrm{hPa}$.

But at Jules Verne's time, probably no really useful measurements about temperature and pressure in those strata of the atmosphere were known; and so I think both boiling temperature and height corresponding to the same pressure can be seen as matching up with the precision that can be expected.

Note that in any case, the pressure is far too low to support human life for a longer time; so unless we assume that oxygen makes up a larger fraction of the Gallian air than it does here on Earth (but there's no statement in the book to really support this supposition), the value given is unrealistically low from a biological point of view. But we'll see later on that it is also unrealistically high from a physical point of view nevertheless. So it may be a good compromise from a fictional point of view as it is.

At [12, ch. I/7, line 1309], Jules Verne describes that Captain Servadac is curious whether a fire will burn in the rarefied air, but notices later that it does indeed without any problems — this may be interpreted as a hint for a higher amount of oxygen, but I think that is fairly far-stretched and thus stick with the assumption that the composition of the air has not changed.

3.2 Barometric Height vs. Stationary Orbit

The scale height used in the classical barometric formula is given as

$$H = \frac{RT}{g}$$

according to [8, p. 164], where R is the specific gas constant (as opposed to Gallia's radius!), T the temperature (assumed to be constant at all heights) and g the gravitational acceleration.

R and T being the same as for the Earth, it is clear that because of diminished gravity according to Section 2.2 the scale height of Gallia will be much larger, namely around 80 km instead of 8 km. This implies that the Gallian atmosphere is "thicker" in some sense (even though the surface pressure is lower).

On the other hand, due to Gallia's faster rotation and also diminished mass the galliastationary orbit is much nearer to the surface as is the geostationary one (the exact altitude can be found by setting Equation 2 to zero (and subtracting the planet's radius of course) — it is 1500 km as compared to the geostationary orbit's altitude of 36000 km).

Taking those two effects together, the question comes to mind what happens when the atmosphere extends up to the galliastationary orbit where the gravity vanishes; obviously, there must not be any hydrostatic pressure there, because no force could counteract it and the gas would be free to escape under its influence.

Of course, there are two important points to take into account: First, the atmosphere does not have any "hard" boundary, but rather the pressure decreases ad infinitum and becomes so small that the whole statistical concept of "pressure" does not make sense anymore above a certain height. I'll consider the above statement in more detail later, but nevertheless it should be quite clear that this height must in any case be well below the stationary orbit.

Second, the gravity from Equation 2 of course only applies if the atmosphere is assumed to precisely follow Gallia's rotation. But below Gallia's exosphere were the pressure has vanished mostly and instead we have to consider individual gas particles, this assumption seems to be fulfilled; as of [12, ch. II/4, line 7309], during the Gallian winter there's a "complete stillness of the atmosphere" which would be disturbed in case of relative rotation between Gallia's surface and some parts of the atmosphere.

In order to work this out more precisely, let's derive a barometric formula which takes into account the dependence $g(z)$ of the gravity acceleration on the height z according to Equation 2. The differential equation that characterizes the pressure p as function of height is (according to [8, p. 164], with $\Phi = gz$)

$$\frac{dp}{dz} = -\frac{gp}{RT}. \tag{4}$$

When we consider T as constant but accept a dependence $g(z)$, Equation 4 can be solved by separation of variables and the solution is the generalized barometric formula:

$$p(z) = p_0 \cdot \exp\left(-\frac{1}{RT}\int_0^z g(\zeta)\, d\zeta\right) \tag{5}$$

In this equation, p_0 is Gallia's surface pressure and z is the height above ground (as opposed to the distance from Gallia's center of gravity!).

Integration of $g(\zeta)$ is easy (albeit the result is somewhat lengthy); using τ for the rotation period (because the T from Equation 2 is already used for the temperature here) and R_G for Gallia's radius, Equation 2 becomes

$$g(\zeta) = \frac{GM}{(R_G+\zeta)^2} - \frac{4\pi^2}{\tau^2}(R_G+\zeta).$$

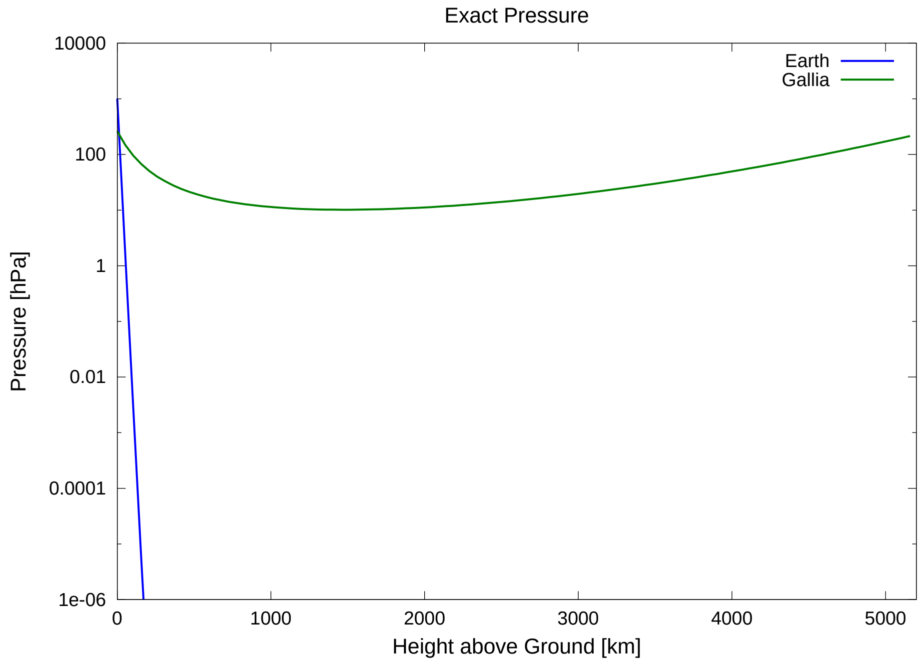

Figure 2: Pressure $p(z)$ according to the generalized barometric formula in Equation 5 for the Earth and Gallia.

Then the definite integral is

$$\int_0^z g(\zeta)\, d\zeta = \frac{GM}{R_g}\left(1 - \frac{1}{1 + \frac{z}{R_G}}\right) - \frac{4\pi^2}{\tau^2}\left(R_G z + \frac{z^2}{2}\right),$$

which completes the barometric formula in Equation 5.

The pressure $p(z)$ from Equation 5 is plotted for the Earth and for Gallia on a logarithmic scale in Figure 2. One can clearly see that the pressure has a minimum at the stationary orbit and then increases again faster than exponentially, going to infinity at far distances. This corresponds to the informal statement above that the atmosphere must "end" well below the stationary orbit.

I assumed an isothermic atmosphere with $T = 0\,°\mathrm{C}$ for those calculations, but the qualitative result is not much changed for other temperatures. For Gallia, the minimum pressure is about 10.2 hPa; this is still quite a large value and so the way Verne depicts it, the atmosphere can never be stable.

Of course, we can play the same game for the Earth. Mathematically, Equation 5 holds all the same; so why has the Earth still an atmosphere? If you consider Figure 2, the Earth's curve goes nearly exponentially down very steeply (which corresponds to the classical barometric formula); and the pressure at the geostationary orbit is $1.5 \cdot 10^{-266}$ hPa, which is small beyond all imagination. Thus for the Earth, while mathematically correct, Equation 5 loses significance well before this height is reached because the pressure becomes so small that all theory of ideal gases that forms the foundation of Equation 4 is not working any longer.

3.3 Thermal Flight

Besides the consideration in Section 3.2, another concept that prohibits small bodies from holding a dense atmosphere is that of thermal flight; see [8, p. 625f]. The idea is that gas particles whose thermal movement exceeds the escape speed can leave the body's gravitational force-field and are lost. Note that in this section, I will always consider only Gallia's gravitational force and neglect the centrifugal component. So here the assumption that the atmosphere takes part in Gallia's rotation is not made; taking this into consideration would make the resulting effects even more severe.

In the case of Gallia, we first have to calculate the escape speed. It is the speed v, such that a particle's kinetic energy equals the potential difference to infinity; in other words, for the escape speed v_∞, the following equation holds:

$$\frac{mv_\infty^2}{2} = \frac{GMm}{r}$$

This can easily be solved for v_∞, and gives the escape speed as

$$v_\infty = \sqrt{\frac{2GM}{r}}, \tag{6}$$

which can also be found at [8, p. 625]. At surface-level on Gallia, $v_\infty = 850\,\frac{\mathrm{m}}{\mathrm{s}}$. Clearly, this is significantly lower than the value of $11200\,\frac{\mathrm{m}}{\mathrm{s}}$ on Earth.

At temperature T, the mean thermal energy of a gas particle is given by $\frac{3}{2}kT$ [8, p. 626]; if the corresponding speed is larger than v_∞, those particles can escape. This is equivalent to the condition

$$\frac{3}{2}kT = \frac{GMm}{r},$$

which gives the critical minimum temperature for this to happen as

$$T = \frac{2}{3}\frac{GMm}{kr}. \tag{7}$$

If the actual gas temperature is at least that large, *blow-off* happens, which is quite an effective way to get rid of the atmosphere according to [8, p. 626].

The critical temperature from Equation 7 depends on the particle's mass m, and thus is different for the distinct gas components forming the atmosphere. In Table 1, the blow-off temperature is given for the Gallian atmosphere at different heights above the ground and for different gases. Depending on which temperature Gallia's exosphere is assumed to be at, some gas components are clearly volatile and can not be hold by Gallia's weak gravitation; but at least the heavier gases may not be susceptible to blow-off.

The actual speeds of gas particles at a certain thermodynamic temperature follow the Maxwell-Boltzmann distribution. According to [14], the corresponding cumulative distribution function is given as

$$F(v) = \operatorname{erf}\left(\frac{v}{\sqrt{2}}\sqrt{\frac{m}{kT}}\right) - \sqrt{\frac{2}{\pi}}\sqrt{\frac{m}{kT}} \cdot v \exp\left(-\frac{mv^2}{2kT}\right).$$

Thus the critical fraction of gas particles that can escape because their speed is larger than v_∞ is

$$f_c = 1 - F(v_\infty). \tag{8}$$

Gas	Molar Mass	Scale-Heights above Ground 0	1	5	10	f_c at $T = 0\,^{\circ}$C
H_2	2	58 K	48 K	28 K	18 K	88.8 %
H_2O	18	521 K	428 K	249 K	163 K	12.6 %
N_2	28	811 K	665 K	387 K	254 K	3.1 %
O_2	32	926 K	790 K	442 K	290 K	1.7 %
Ar	40	1158 K	950 K	553 K	363 K	0.5 %
CO_2	44	1274 K	1045 K	608 K	399 K	0.3 %

Table 1: Escape characteristics of different gases in the Gallian atmosphere based on Equation 7 and Equation 8.

Taking Equation 6 and Equation 8 together, the fractions in Table 1 are found. While the heavier gases have rather low fractions, they are also clearly distinct from zero and thus even those will gradually escape.

Overall, the numbers in Table 1 indicate that also from the point of view of thermal flight, Gallia's atmosphere is not stable and can not exist in the way as described by Jules Verne. The only possibility is that flight is rather slow and the atmosphere can survive at least over the two years it has to for the story, even though it gets thinner during that time.

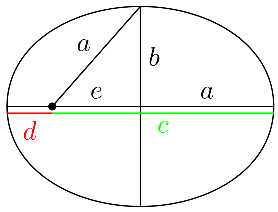

Figure 3: Elliptical orbit with semi-axes and perihelion / aphelion.

4 Gallia's Orbit

It is well known that the mechanics of the planets (and other bodies, in particular Gallia) revolving around the Sun can be very well approximated by movement of those bodies within the Sun's gravitational force-field (because the Sun's mass is much larger than that of any of the planets, and especially Gallia, we can assume the field to be constant and consider just Gallia moving within it without disturbing it). Neglecting the influences of planets onto each other is also (usually) justified for approximative models (and I will do so).

Under these assumptions, possible trajectories are conic sections — ellipses (including circles as special case) for negative total energy, parabolas and hyperbolas for non-negative energy. According to [12, ch. II/3, line 7229], in the case of Gallia it is an ellipse.

4.1 Perihelion and Aphelion

Gallia reaches its perihelion on Jan, 15th, 1 (according to [15, ch. I/15]). It is somewhere inside of Venus' orbit, but the exact position is never stated. The aphelion however is specified as $220 \cdot 10^6\,\mathrm{l}$ and reached on Jan, 15th, 2 [15, ch. II/13]. Additionally, the orbit's circumference is given as "little over" $U = 630 \cdot 10^6\,\mathrm{l}$ as of [15, ch. II/11].

Let c and d be the aphelion and perihelion distances for Gallia's elliptical orbit, let a and b denote the major and minor semi-axes and let e (with $a^2 = b^2 + e^2$ according to Pythagoras) be the linear eccentricity. Then (as illustrated by Figure 3) those values are related by

$$\left\{ \begin{array}{rcl} c + d & = & 2a \\ c & = & a + e. \end{array} \right. \tag{9}$$

Thus when given the perihelion and aphelion distances as d and c, we can find the elliptical orbit's axes according to Equation 9 as $a = \frac{c+d}{2}$, $e = c - a$ and $b = \sqrt{a^2 - e^2}$. Unfortunately, we have to deal with the ellipse's circumference (because this is a piece of data given in the book) which is a quantity that can not be written in closed form as expression of a and b (and thus perihelion and aphelion distances) — but there exist approximation formulas that can be applied to calculate the circumference with much better precision than the round number given anyways, so this is no practical problem.

Clearly, the unknown perihelion distance has to be a number between zero and the aphelion. Assuming "near" zero, the orbit would degenerate to a line and thus the circumference would be twice the aphelion distance in this case, namely $U_1 = 440 \cdot 10^6\,\mathrm{l}$ which is less than the known circumference. For the other extreme, a circular orbit with $a = b = c = d$, the circumference would be $U_2 = 2\pi \cdot c = 1382 \cdot 10^6\,\mathrm{l}$ which is too large. Because the circumference as function of perihelion distance is obviously continuous and strictly increasing, it follows by the intermediate value theorem that there's a uniquely defined perihelion distance in-between those two boundaries that matches Jules Verne's circumference U — this can, for instance, be calculated numerically via an interval bisection approach. This gives the perihelion distance as

$$d = 25.5 \cdot 10^6\,\mathrm{l} = 0.68\,\mathrm{AU} = 0.94 \times \text{mean distance Venus} - \text{Sun},$$

which is a very reasonable value and matches with the qualitative description in the book, namely that it is somewhere little inside Venus' orbit. As already discussed above, the axes a and b of Gallia's orbit follow then directly, which means that its geometry is now fully known. Namely,

$$\begin{aligned} a &= \frac{c+d}{2} = 123 \cdot 10^6\,\mathrm{l} \\ b &= \sqrt{a^2 - (c-a)^2} = 75 \cdot 10^6\,\mathrm{l}. \end{aligned}$$

4.2 Impossible Schedule

Kepler's third law establishes a connection between size of a planetary orbit (in particular its major semi-axis a) and the corresponding period duration T, which is (see [4, p. 88])

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}, \tag{10}$$

where M is the Sun's mass; the planet's mass is (as before) neglected in respect to it. Of course, this relation has to be fulfilled by Gallia, too.

In this case, we already know $a = 123 \cdot 10^6\,\mathrm{l}$ and the orbital period is specified as $T = 2\,\mathrm{a}$ (exactly two terrestrial years) — see [12, ch. II/3, line 7244] or [15, ch. II/4]. Unfortunately, these two values clearly do not satisfy Equation 10, not even approximately.

While Kepler's third law is not mentioned explicitly in the book (like the second law and in some sense also the first is), this is something I expect Verne to know about without any doubt. My interpretation is that he wanted a large orbit so as to bring Gallia's population deeper into space while he did not want them to be as long in the cold outer regions during their travelings as they must necessarily be for his planned orbital size.

Solving Equation 10 for either T or a allows us to find the "correct" period or semi-axis for the stated orbit or two years' journey, respectively. Insisting on $T = 2\,\mathrm{a}$, we find

$$a = \sqrt[3]{T^2 \frac{GM}{4\pi^2}} = 59 \cdot 10^6\,\mathrm{l}$$

and further for this orbit — assuming the perihelion is unchanged — that its aphelion must be $c = 93 \cdot 10^6\,\mathrm{l} = 2.49\,\mathrm{AU}$ from the Sun. This is just beyond Mars' orbit instead of Jupiter's.

On the other hand, assuming that the proper orbit as stated in the book was more important to Jules Verne than his claimed period, the correct duration of

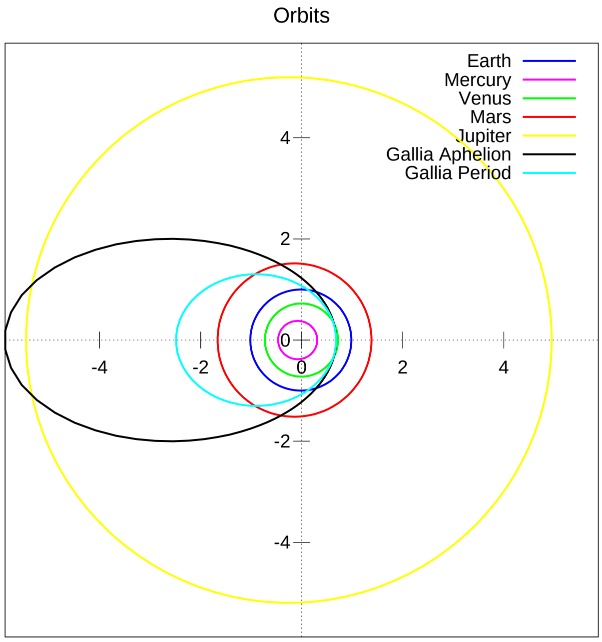

Figure 4: Gallian orbit possibilities compared to planets. Units are AU.

the Gallians' journey is

$$T = \sqrt{a^3 \frac{4\pi^2}{GM}} = 5.96\,\mathrm{a},$$

which is much longer than his two years. My opinion is that this second version is probably what Verne had in mind; for later analysis, I assume that his stated distances are correct but the time must be scaled by a factor of (nearly) 3 to match up — that is, I will divide the theoretically correct time scale by this factor in order to get results comparable to the stated values; i.e., for a full orbit the time-axis should actually cover months 0–$6 \cdot 12$, but I will scale it down so that the full orbit takes place in the months 0–$2 \cdot 12$ only and use Verne's original value-time pairs as comparison.

Figure 4 schematically depicts the orbits of the five innermost planets (up to Jupiter) together with the two possibilities of Gallia's orbit. This plot reflects what we already found out, namely that Gallia's perihelion lies just within Venus' orbit and that — while the stated aphelion is outside of Jupiter's — the one matching a two year period is only beyond Mars'.

4.3 Distance from Sun

Gallia's orbit is the solution to the classical Kepler problem, i.e., movement within the Sun's gravitational force-field with potential given as $V(r) = -\frac{GMm}{r}$.

This problem is very well analyzed in general and discussed, for instance, in [4, p. 83ff]; note that the potential there is expressed more generally as

$V(r) = -\frac{k}{r}$ so that in our case $k = GMm$. The trajectory in polar coordinates (r, ϕ) is given as

$$r(\phi) = \frac{p}{1 + \epsilon \cdot \cos\phi} \tag{11}$$

with the two constants

$$\begin{aligned} p &= \left(\frac{L}{m}\right)^2 \frac{1}{GM} \\ \epsilon^2 &= 1 + \frac{E}{m}\left(\frac{L}{m}\right)^2 \frac{2}{G^2M^2}, \end{aligned}$$

where E is the total energy of Gallia (kinetic and in the gravitational potential) and L is its (orbital) angular momentum, both of which are conserved quantities during orbital movement.

When we choose the $\phi = 0$ axis of the polar coordinates accordingly and know the aphelion and perihelion distances c and d of the orbit, we can calculate p and ϵ by solving the linear system

$$\begin{cases} p - d \cdot \epsilon &= d \\ p + c \cdot \epsilon &= c \end{cases}$$

that stems from Equation 11 for $\phi = 0$ and $\phi = \pi$, respectively. Note that ϵ is also the numerical eccentricity of the orbit's ellipse and $\epsilon = \frac{e}{a}$ holds for the linear eccentricity e and the major semi-axis a that could be found differently from perihelion and aphelion (based on Equation 9).

If we want not only the curve but also $r(t)$ and $\phi(t)$ as functions of time, we can consider the differential equation

$$mr^2\dot{\phi} = L \tag{12}$$

(see [4, p. 67]) or equivalently

$$\dot{\phi} = \frac{L}{m}\frac{1}{r(\phi)^2}.$$

By choosing, for instance, $\phi(0) = 0$, we arrive at an initial value problem that can be integrated numerically to find $\phi(t)$ and then by Equation 11 $r(t)$, if only the constant $\frac{L}{m}$ is known. This quantity can be found from p which in turn is known from the orbit specification with perihelion and aphelion distances.

Alternatively, consider Equation 12. In the form

$$\frac{L}{m} = r^2\dot{\phi} = 2\frac{dA}{dt} \tag{13}$$

this is just Kepler's second law about equal areas and if we integrate the equation over one orbit we get the full ellipse's area on the right-hand side:

$$\frac{L}{m} \cdot T = 2\pi ab$$

The orbital period T is known from Kepler's third law (and was already calculated), so we find for the unknown constant:

$$\frac{L}{m} = \frac{2\pi ab}{T} \tag{14}$$

Date	Distance	At Planet	Reference	
Jan, 1st, 1		Earth	[15, ch. II/2]	[12, ch. II/2, line 6849]
Jan, 10th, 1		Venus	[15, ch. II/4]	[12, ch. II/3, line 7146]
Jan, 20th, 1		Venus		[12, ch. I/8, line 1767]
Feb, 1st, 1		Earth	[15, ch. II/4]	[12, ch. II/3, line 7148]
Feb, 13th, 1		Mars	[15, ch. II/4]	[12, ch. II/3, line 7149]
Feb, 15th, 1	$59 \cdot 10^6\,\mathrm{l}$		[15, ch. I/15]	[12, ch. I/15, line 3649]
Mar, 1st, 1	$78 \cdot 10^6\,\mathrm{l}$		[15, ch. I/17]	[12, ch. I/17, line 4213]
Apr, 30th, 1	$110 \cdot 10^6\,\mathrm{l}$		[15, ch. II/5]	[12, ch. II/4, line 7379]
May, 31st, 1	$139 \cdot 10^6\,\mathrm{l}$		[15, ch. II/5]	[12, ch. II/4, line 7405]
Jun, 30st, 1	$155 \cdot 10^6\,\mathrm{l}$		[15, ch. II/5]	[12, ch. II/4, line 7501]
Jul, 31st, 1	$172 \cdot 10^6\,\mathrm{l}$		[15, ch. II/6]	[12, ch. II/5, line 7720]
Aug, 31st, 1	$197 \cdot 10^6\,\mathrm{l}$		[15, ch. II/9]	[12, ch. II/8, line 8461]
Sep, 1st, 1		Jupiter		[12, ch. II/8, line 8607]
Dec, 15th, 1	$216 \cdot 10^6\,\mathrm{l}$		[15, ch. II/11]	
Jan, 15th, 2	$220 \cdot 10^6\,\mathrm{l}$		[15, ch. II/13]	[12, ch. II/12, line 9806]
Jun, 1st, 2	$197 \cdot 10^6\,\mathrm{l}$		[15, ch. II/14]	[12, ch. II/13, line 10028]
Nov, 30th, 2	$78 \cdot 10^6\,\mathrm{l}$		[15, ch. II/17]	[12, ch. II/16, line 11127]
Dec, 15th, 2		Mars	[15, ch. II/17]	[12, ch. II/16, line 11180]
Dec, 31st, 2	$40 \cdot 10^6\,\mathrm{l}$		[15, ch. II/18]	
Jan, 1st, 3		Earth		

Table 2: Gallia's distances from Sun as claimed by Jules Verne.

These results enable us to calculate the theoretical orbit, most notably the distances from the Sun $r(t)$ as function of time, for Gallia (assuming the perihelion and aphelion determined in Section 4.1).

On the other hand, Jules Verne also makes a lot of statements about Gallia's distance from the Sun at certain dates. It is not always perfectly clear to which exact day some distances belong, but I think in all cases it can be inferred quite unambiguously. Sometimes instead of a distance the position relates to a planetary orbit (like "Gallia crossed the orbit of Mars"), in which case I assumed it to be the mean distance of that planet from the Sun. Those claimed distances are summarized in Table 2.

It is evident that the distances from Sun are "symmetrical" in the sense that time t before reaching the aphelion they are the same as t after reaching it. In Figure 5, I compare Verne's claimed values, his values when mirrored around the aphelion in this way and the theoretical curve (with time scaled to match his two year period). While he hit the correct curve near the perihelion and aphelion quite well, the values in-between are notably off; they seem to nearly follow a straight line, so maybe he did interpolate linearly there (but that's only a guess). On the other hand, he was clearly aware of the temporal symmetry aspect. Actually, consulting Table 2 one finds that some stated values are given identically at dates that are symmetric around the aphelion on Jan, 15th, 2. For instance, the orbit of Mars was crossed one month after as well as one month before the perihelion; the distance of $78 \cdot 10^6\,\mathrm{l}$ from Sun reached one and a half month after as well as before the perihelion. Verne seemed to calculate in months, though, and some minor differences in the plot can be explained by different lengths of months when using days or seconds based on the calendar.

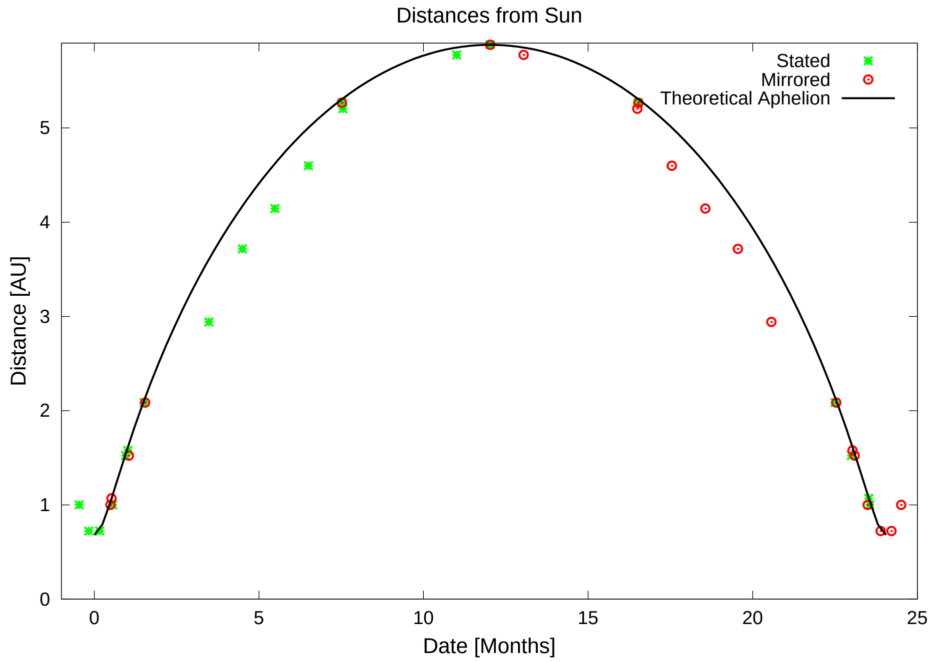

Figure 5: Gallia's distance from Sun as stated and calculated.

4.4 Orbital Speed

In addition to the distances of Gallia from the Sun, it is interesting to consider the orbital speed at different times. It is clear that the speed is higher near the perihelion and smaller near the aphelion.

For Gallia's movement, the total energy E is conserved, and from Equation 11 we get the relation

$$\epsilon^2 = 1 + \frac{E}{m}\left(\frac{L}{m}\right)^2 \frac{2}{G^2M^2}$$

which results in

$$\frac{E}{m} = \frac{\epsilon^2 - 1}{2} G^2M^2 \left(\frac{L}{m}\right)^{-2}.$$

Therefore, because the orbit's eccentricity ϵ is known from statements in the book and also the constant $\frac{L}{m}$ can be calculated from them (e.g., Equation 14 or Section 4.3 in general), we can consider $\frac{E}{m}$ as known quantity.

However, the energy is also given as sum of kinetic and potential energy, namely

$$E = \frac{mv^2}{2} + V(r) = \frac{mv^2}{2} - \frac{GMm}{r}.$$

Because E and also $\frac{E}{m}$ is constant (does not depend on the current distance r from the Sun) and only the orbital speed v changes for different r, this equation

Interval		Advance	Reference	
Jan, 1st – Jan, 31st, 1	?	$82 \cdot 10^6\,\mathrm{l}$	[15, ch. I/15]	[12, ch. I/15, line 3581]
Feb, 1st – Feb, 29th, 1	?	$59 \cdot 10^6\,\mathrm{l}$	[15, ch. I/17]	[12, ch. I/17, line 4214]
Apr, 1st – Apr, 30th, 1		$39 \cdot 10^6\,\mathrm{l}$	[15, ch. II/5]	[12, ch. II/4, line 7378]
May, 1st – May, 31st, 1		$30.4 \cdot 10^6\,\mathrm{l}$	[15, ch. II/5]	[12, ch. II/4, line 7404]
Jun, 1st – Jun, 30th, 1		$27.5 \cdot 10^6\,\mathrm{l}$	[15, ch. II/5]	[12, ch. II/4, line 7501]
Jul, 1st – Jul, 31st, 1		$22 \cdot 10^6\,\mathrm{l}$	[15, ch. II/6]	[12, ch. II/5, line 7720]
Aug, 1st – Aug, 31st, 1		$16.5 \cdot 10^6\,\mathrm{l}$	[15, ch. II/9]	[12, ch. II/8, line 8461]
Sep, 1st, 1 – Jan, 15th, 2		$81 \cdot 10^6\,\mathrm{l}$	[15, ch. II/9]	[12, ch. II/8, line 8463]
Dec, 1st – Dec, 31th, 1	?	$11.5 \cdot 10^6\,\mathrm{l}$	[15, ch. II/11]	[12, ch. II/10, line 9094]
Jul, 1st – Aug, 31st, 2		$164 \cdot 10^6\,\mathrm{l}$	[15, ch. II/14]	[12, ch. II/13, line 10055]
Nov, 1st – Nov, 30th, 2		$59 \cdot 10^6\,\mathrm{l}$	[15, ch. II/17]	

Table 3: Gallian advance during certain time intervals. Both dates are meant inclusively, so the duration of one such interval is one day longer than the difference of end- and start-date.

gives us the relation $v(r)$ between distance r and orbital speed at that distance:

$$v(r) = \sqrt{2\left(\frac{E}{m} + \frac{GM}{r}\right)} \tag{15}$$

Putting Equation 15 and the numerical solution for $r(t)$ from Section 4.3 together gives the theoretically expected orbital speed $v(t)$ for all times during the journey.

In the book, there are no really explicit statements about the orbital speed (except for few positions, not relating to Gallia's speed changing with distance from Sun). But there are figures about how far Gallia advances along its orbit during a certain interval of time, like a specified month; with those values, it is at least possible to find a mean speed during that interval, which can still be compared to the theoretical value. This is summarized in Table 3, but once again the dates are often ambiguously stated and I've marked some intervals as "not sure" there with question marks.

As with the distances from Sun, the speeds should also be symmetric in time around the aphelion. Figure 6 shows a comparison of theoretical prediction with mean speeds calculated from this data as well as the same values mirrored in time. The theoretical curve is scaled in time as to fit into 24 months. In addition, the plot contains the "correct" speeds as of Equation 15 for the distances from Sun claimed by Jules Verne (see Table 2).

As before, Verne did not predict the curve precisely — he did reflect the fact that the distance decreases towards the aphelion qualitatively correct, though. In the middle parts, as with the position his values do not follow the curve but rather "cut it short". However, in relation to the stated positions, his speed predictions are more or less accurate (so the wrong speed there may be due to his earlier errors in position), and near the orbit's extremal points, he even got it quantitatively right — especially the speed at Gallia's aphelion is correct with a very good precision! Taking into account that the value plotted is only the mean over a full month, the same is also true for the speed at the very beginning.

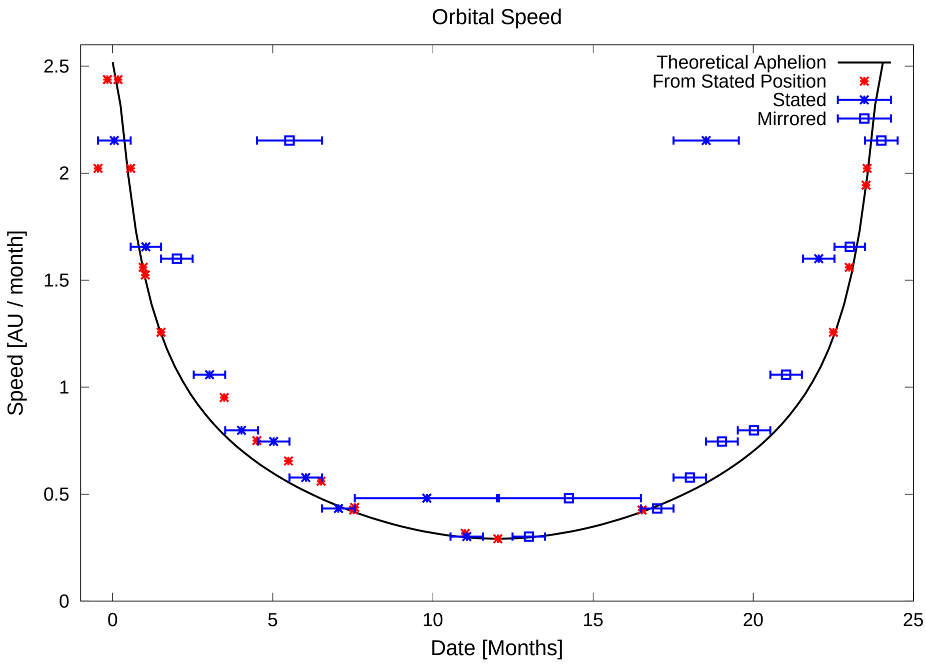

Figure 6: Theoretical and claimed orbital speeds. "Error-bars" mark the time interval over which the speed represents the mean.

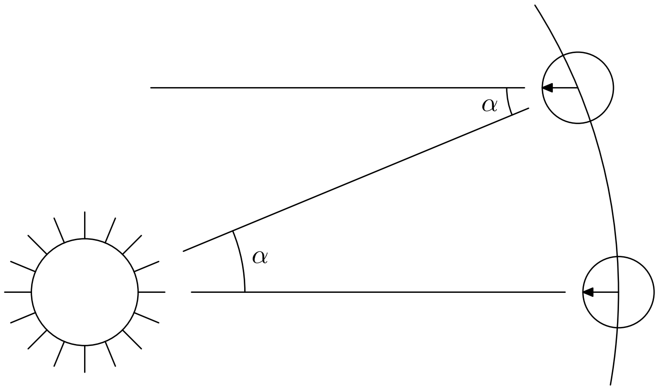

Figure 7: Relative position after one sidereal day.

Unfortunately, here he still seemed not to take that much care about his numbers over all; the statement about Sep, 1st, 1 – Jan, 15th, 2 is quite out of order, and the stated travel distance of $164 \cdot 10^6\,\mathrm{l}$ between Jul, 1st – Aug, 31st, 2 is obviously tremendously wrong (which is clearly visible in Figure 6, too).

4.5 Day-Length Differences

An interesting phenomenon for inhabitants of a body that revolves in an eccentric orbit around the Sun (just as Gallia does) rather than an almost circular is that by Kepler's second law the angular orbital speed is lower near the aphelion and higher near the perihelion while the body's own rotation stays (except for changes in the momentum of inertia) constant — this means that while the sidereal day-length is of course fixed (conservation of angular momentum), the synodical days vary in length over the course of one year.

Gallia's rotation is retrograde, but according to [12, ch. II/3, line 7046] also Gallia's orbital movement was retrograde before the collision with the Earth; I think it's reasonable to assume thus that its later movement is also retrograde, so that it is of the same orientation as Gallia's rotation (if it is not, then all day-length differences have opposite sign to my results later on, but otherwise it stays the same).

Thus during the course of one sidereal rotation (that is, exactly 360° around its own axis) it has already moved a little along its orbit, so that some further rotation is needed to align it in the same position relative to the Sun — a synodical day is slightly longer than a sidereal, and the difference depends on the angle traversed on its orbit during one sidereal rotation (which has a fixed length all the time; within less than a percent of precision it can be approximated by the day-length stated in the book of $T = 12\,\mathrm{h}$ that is probably the synodical length near Gallia's perihelion). This situation is sketched in Figure 7, the angle α is the angular change in orbital position during one sidereal day.

So during one *synodical* day, Gallia has to rotate $360° + \alpha$ in order to compensate for this, and thus the difference between synodical and sidereal day-length will be

$$\Delta T = \frac{\alpha}{2\pi} \cdot T \tag{16}$$

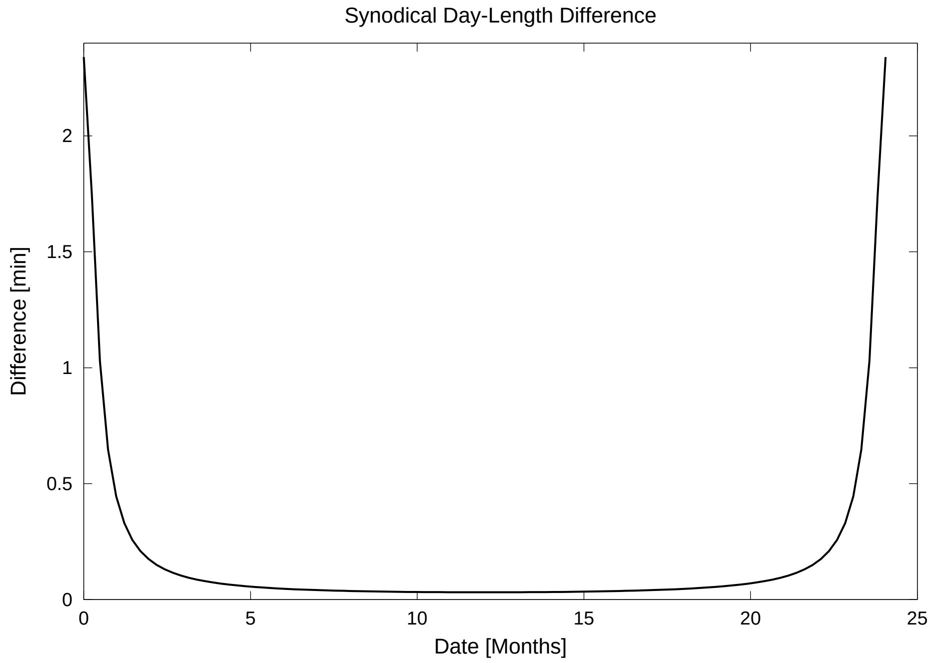

Figure 8: Difference between synodical and sidereal days during Gallian year.

neglecting the small distance Gallia advances further on the orbit during that time, too.

According to Equation 12, the time-derivative of Gallia's angular position is $\dot{\phi}(r) = \frac{L}{m}\frac{1}{r^2}$ and so $\alpha(r) \approx \frac{L}{m}\frac{1}{r^2} \cdot T$. We already calculated the distance from Sun $r(t)$ along Gallia's journey in Section 4.3, so with Equation 16 we can now find the day-length difference $\Delta T(t)$ (compared to Gallia's sidereal day) over the course of its orbit, which is plotted in Figure 8.

We see that the synodical days (as experienced by the Gallians between two sunrises) differ in length more than 2 min between the perihelion and a few months after that event. This effect is never explicitly mentioned in the book and should probably be noticeable with careful observation, but I believe that it could as well also be missed easily, so do not blame Jules Verne or his characters for not mentioning or even observing it.

4.6 Geometry at Odds

When combining the data given in Table 2 and Table 3, one finds that Verne has given some triplets of distance from Sun at time t_1, traveled from t_1 to t_2 and distance at time t_2. The traveled distance corresponds to an elliptical arc, and together with the two distances from the Sun it forms a nearly triangular shape (at least in good approximation if the time interval is rather short compared to one full orbit).

From Kepler's second law (or more precisely, Equation 13) we know that its area should be proportional to the time interval $t_2 - t_1$, and that we can

From	To	Distance 1	Traveled	Distance 2	$\frac{L}{m}$
Jan, 1st	Jan, 31st, 1	1 AU	$82 \cdot 10^6$ l	1 AU	-
Feb, 1st	Feb, 29th, 1	1 AU	$59 \cdot 10^6$ l	$78 \cdot 10^6$ l	$1.4 \cdot 10^{16}\ \frac{\mathrm{m}^2}{\mathrm{s}}$
May, 1st	May, 31st, 1	$110 \cdot 10^6$ l	$30.4 \cdot 10^6$ l	$139 \cdot 10^6$ l	$0.7 \cdot 10^{16}\ \frac{\mathrm{m}^2}{\mathrm{s}}$
Jun, 1st	Jun, 30th, 1	$139 \cdot 10^6$ l	$27.5 \cdot 10^6$ l	$155 \cdot 10^6$ l	$2.0 \cdot 10^{16}\ \frac{\mathrm{m}^2}{\mathrm{s}}$
Jul, 1st	Jul, 31st, 1	$155 \cdot 10^6$ l	$22 \cdot 10^6$ l	$172 \cdot 10^6$ l	$1.4 \cdot 10^{16}\ \frac{\mathrm{m}^2}{\mathrm{s}}$
Aug, 1st	Aug, 31st, 1	$172 \cdot 10^6$ l	$16.5 \cdot 10^6$ l	$197 \cdot 10^6$ l	-
Sep, 1st, 1	Jan, 15th, 2	$197 \cdot 10^6$ l	$81 \cdot 10^6$ l	$220 \cdot 10^6$ l	$2.1 \cdot 10^{16}\ \frac{\mathrm{m}^2}{\mathrm{s}}$

Table 4: $\frac{L}{m}$ via approximate triangle areas.

calculate the conserved quantity $\frac{L}{m}$ that is characteristic to Gallia's orbit from the area corresponding to a given time.

When we approximate the shape with a triangle, we know all three of its sides a, b and c as the two distances from Sun and the distance traveled in-between; then with Heron's formula, the triangle's area is given as

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$ is the triangle's semi-perimeter.

With this, the corresponding values of $\frac{L}{m}$ for each possible triplet are given in Table 4. Note that the dates corresponding to distances in Table 2 are not given very exactly, so we have to allow for them to vary up to one day, and especially it is most of the time not clear whether a distance corresponds to the last day of a month or the first of the following.

It is easy to see that also this point of view shows that Verne was not very careful with speed numbers (because "correctly", all $\frac{L}{m}$ values should be the same — but at least some match quite good). However, in two cases (the ones where no $\frac{L}{m}$ is given) his statements violate even the triangle inequality! In the very first row, $1\,\mathrm{AU} = 37.5 \cdot 10^6\,\mathrm{l}$ and thus $2\,\mathrm{AU} < 82 \cdot 10^6\,\mathrm{l}$ which means that the claimed numbers can not possibly be correct, because when traveling $82 \cdot 10^6\,\mathrm{l}$ from a position 1 AU from the Sun, Gallia must in any case end up more than 1 AU distant from it. But as I marked this travel-interval as unclear in Table 3, this may also result from an incorrectly extracted number.

In the second case however, the statements in the book are quite clear. And here again, when Gallia only travels $16.5 \cdot 10^6\,\mathrm{l}$ during the month of August, it can at most end up being $188.5 \cdot 10^6\,\mathrm{l}$ from the Sun at the end, as compared to the stated $197 \cdot 10^6\,\mathrm{l}$!

This is of course something that is clear just with common sense and without anything related to "physics", so Jules Verne seemed to have given not much thought and effort to those numbers... Or it is a typographic error, because there are also some inconsistencies between [15] and [12] (although the unit is the same in the translation in those cases, so this is not related to incorrect conversion) and even within one text where a number is given twice (see [15, ch. I/15] where the traveled distance is both stated as $32 \cdot 10^6\,\mathrm{l}$ and $82 \cdot 10^6\,\mathrm{l}$).

5 Temperature Modeling

While Gallia's axis of rotation is not inclined with respect to its orbital plane and hence there are no seasons in the classical sense on Gallia, because of its very eccentric elliptical orbit there are obviously strong temperature differences over the course of one Gallian year. Near the perihelion, Gallia is less than 1 AU distant from the Sun and receives thus more light and heat than the Earth (about as much as Venus does), but near the aphelion it is considerably less at more than 5 AU distance.

This is also a motive addressed in the book, and in order to escape the cold during what Verne calls the "Gallian winter", the inhabitants find shelter near an active volcano. Jules Verne again has a lot of explicit temperature values in the book, and in this section I will use a (very simple) model to check those.

5.1 Radiation Intensity

While radioactive decay produces heat to some degree on the Earth (and other planets as well of course), the by far most important source of heat (and energy in general) is absorption of the Sun's radiation.

The same is evidently true for Gallia — so as a first step, we have to consider in what relation this quantity is to Gallia's distance from the Sun. Because the amount of radiation that passes through concentric spheres erected around the Sun's center must always be the same (corresponding to the luminosity of the Sun) regardless of the sphere's radius, the radiation intensity must be inversely proportional to r^2 (where r is the distance from the Sun) because the spheres' surface areas increase in that proportion.

The power of radiation passing through $1\,\mathrm{m}^2$ orthogonal to the rays at 1 AU distance is also referred to as the *solar constant* S. The actual value of S is changing with solar activity, but I will not use it later on anyways.

More important is that the "solar constant" $S(r)$ for a different distance is (by the above consideration) given as

$$S(r) = \frac{r_0^2}{r^2} \cdot S_0 \tag{17}$$

with S_0 being a "reference value" at distance r_0, e.g., $S_0 = S$ and $r_0 = 1\,\mathrm{AU}$.

According to [12, ch. II/8, line 8552], "the amount of light and heat received" by Jupiter is only $\frac{1}{25}$ of that received by the Earth — with Jupiter's average distance from the Sun being very roughly 5 AU. This fits perfectly with Equation 17, so Jules Verne clearly knew about the correct relation in this respect.

5.2 Radiation Balance

A very simple method to model the mean temperature of Gallia as a whole is by considering input and output flux of heat; see also [8, p. 463ff]. Because space surrounding Gallia is of course non-conducting, the only way this can happen is via absorption and emission of radiation.

The incoming radiation is given via the solar constant and the area Gallia covers when seen from the Sun — this is a circle with the same radius R as Gallia. Not all of this radiation is absorbed, however, because a certain part is immediately reflected. This fraction is called the albedo A. Thus, the incoming heat is given by:

$$Q_{\text{in}} = (1 - A) \cdot S(r) \cdot \pi R^2 \tag{18}$$

On the other hand, by the Stefan-Boltzmann law Gallia also emits heat in form of infrared light itself, depending on its temperature and over all its surface area. We also have to consider the greenhouse-effect, namely that Gallia's atmosphere — probably similar to the Earth's — does in turn absorb a certain part of this radiation and re-emits it back to the surface. Let ϵ be Gallia's emissivity as a "black body" and τ the fraction of heat radiation that actually escapes through the atmosphere. Then the emitted heat at temperature T is

$$Q_{\text{out}} = 4\pi R^2 \cdot \epsilon\tau\sigma \cdot T^4. \tag{19}$$

Note that without the Stefan-Boltzmann law it would not be clear how to join the concepts of Gallian surface temperature and radiation received from the Sun! This plays an essential role here, but was not yet known at the time when Jules Verne published his novel.

If we neglect all further flows of heat and want to find the *equilibrium temperature* at a certain distance r from the Sun, those two flows must equal each other: $Q_{\text{in}} = Q_{\text{out}}$. Taking Equation 18 and Equation 19 together and solving for T, this gives

$$T(r) = \sqrt[4]{S(r)\frac{1 - A}{4\epsilon\tau\sigma}}, \tag{20}$$

where $\frac{1-A}{4\epsilon\tau\sigma}$ as well as the solar constant are still unknown (they could be estimated based on the descriptions in the book, but this would probably be only correct to some degree).

If we use Equation 20 for some reference point at distance r_0 with known equilibrium temperature T_0 and take Equation 17 into consideration, we can get rid of those unknowns:

$$T(r) = \sqrt[4]{\frac{r_0^2}{r^2} S_0 \cdot \frac{1 - A}{4\epsilon\tau\sigma}} = \sqrt{\frac{r_0}{r}} \cdot T_0$$

So the temperature scales with the square root of the distance r.

5.3 Temperature of Outer Space

While the argument and derivation in Section 5.2 should work quite well as rough approximation, an interesting point of historical consideration is that Jules Verne mentions not only in his current work [12, ch. I/16, line 3840] but also, for instance, in his novel [11] that Fourier estimated the temperature of outer space to be about −60 °C. While this exact number is not stated there, in [3] the temperature of outer space is mentioned as probably "little below that of the polar regions". According to Fourier, this constant temperature of every point in space is caused by the radiation of the vast multitude of stars in the universe — so this idea is in some sense quite similar to the modern notion of the cosmic microwave background.

While Fourier's temperature is of course wrong, I think it prudent to repeat the modeling of Gallia's temperature under the assumption that Fourier was correct; after all, this is also the impression that Jules Verne had.

Date	Temperature	Reference	
Jan, 15th, 1	50 °C	[15, ch. I/8]	[12, ch. I/8, line 1580]
Feb, 11th, 1	15 °C–20 °C	[15, ch. I/12]	[12, ch. I/12, line 2538]
Feb, 25th, 1	−2 °C	[15, ch. I/17]	[12, ch. I/17, line 3999]
Mar, 6th – 10th, 1	−6 °C	[15, ch. I/20]	[12, ch. I/20, line 5028]
Mar, 20th, 1	−8 °C	[15, ch. I/21]	[12, ch. I/21, line 5438]
Mar, 26th, 1	−12 °C	[15, ch. I/23]	[12, ch. I/23, line 5787]
Mar, 26th – Apr, 1st, 1	−16 °C	[15, ch. I/23]	[12, ch. I/23, line 5837]
Apr, 15th, 1	−22 °C	[15, ch. I/24]	[12, ch. I/24, line 6049]
Apr, 23rd – May, 12th, 1	−30 °C	[15, ch. II/5]	[12, ch. II/4, line 7302]
Dec, 20th, 1	−53 °C	[15, ch. II/12]	[12, ch. II/11, line 9245]
Oct, 1st, 2	−35 °C–−30 °C	[15, ch. II/15]	[12, ch. II/14, line 10123]
Nov, 1st, 2	−12 °C–−10 °C	[15, ch. II/16]	[12, ch. II/15, line 10487]
Dec, 1st, 2	0 °C	[15, ch. II/17]	

Table 5: Temperature statements in the book.

Let $\overline{T} = -60\,°\mathrm{C}$ be the background temperature. In order to adapt the model in Section 5.2, we have to add an additional inward flow of heat $\overline{Q}$ such that without the Sun's radiation, T would be $\overline{T}$ instead of 0. This flow must be just as large as to compensate the temperature emission at $\overline{T}$, namely

$$\overline{Q} = 4\pi R^2 \cdot \epsilon\tau\sigma \cdot \overline{T}^4,$$

so that it weighs up to Q_{out} from Equation 19 at this temperature.

Then the balance becomes $Q_{\text{in}} + \overline{Q} = Q_{\text{out}}$ and solving for T in this new setting results in

$$T(r) = \sqrt[4]{S(r)\frac{1-A}{4\epsilon\tau\sigma} + \overline{T}^4} \tag{21}$$

as extended version of Equation 20. As before, I want to eliminate the unknown constants by introducing a reference point of temperature T_0 at distance r_0.

Equation 21 at this point implies

$$T_0^4 = S_0\frac{1-A}{4\epsilon\tau\sigma} + \overline{T}^4 \quad \Leftrightarrow \quad S_0\frac{1-A}{4\epsilon\tau\sigma} = T_0^4 - \overline{T}^4$$

and so together with Equation 17, the equilibrium temperature at distance r adapted for Fourier's background radiation is

$$T(r) = \sqrt[4]{\frac{r_0^2}{r^2}\left(T_0^4 - \overline{T}^4\right) + \overline{T}^4}.$$

5.4 Putting it to the Test

In Table 5, the temperature values specified in the book are summarized. As before, there are some uncertainties with the dates in some places, so those may not be considered accurate to a single day; but it should be precise enough for a rough comparison with the models developed in Section 5.2 and Section 5.3. In the cases where a time interval is given, the stated temperature falls somewhere within it, but it is not clear, when exactly.

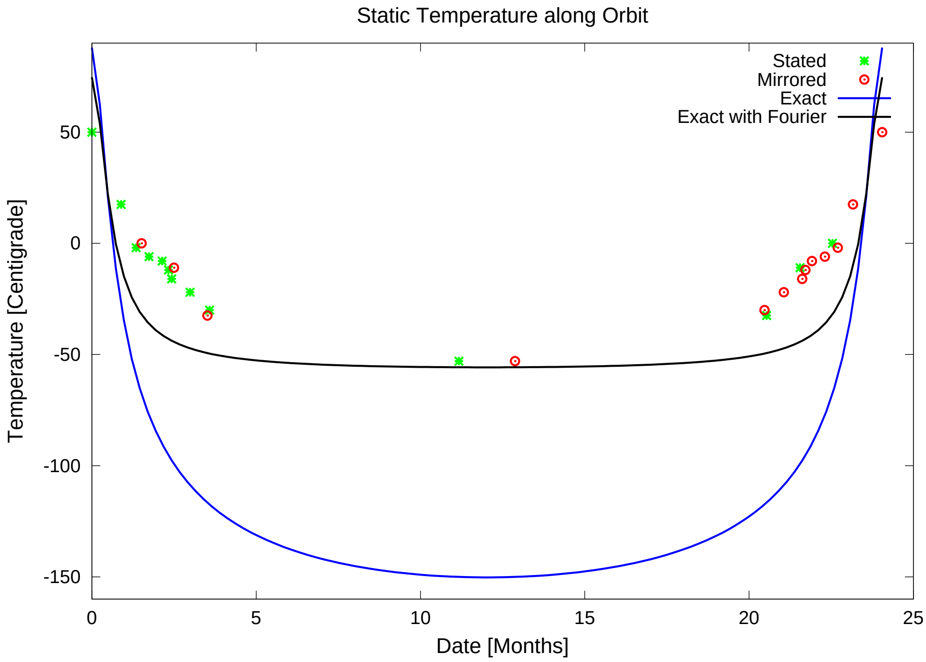

Figure 9: Equilibrium temperatures and Jules Verne's values.

Figure 9 shows a plot of those values together with the temperatures predicted by the above models (i.e., with and without consideration of Fourier's space temperature), where the distance $r(t)$ from the Sun is taken from Section 4.3.

As reference point I chose 25 °C at 1 AU distance, which is little above the value stated for Feb, 11th, 1. Earth's orbit was crossed on the 1st according to Table 2, so this seems reasonable to me.

Once again, for the equilibrium temperature as I considered, we can mirror the values in time around the aphelion, this is also done in the figure. It seems that Jules Verne considered it that way, too, as his values match up closely to the mirrored ones. The real temperature around the aphelion is with $-150\,°\mathrm{C}$ far lower than what Jules Verne anticipated and also lower than Fourier's "minimum" temperature, but assuming Fourier to be right, Jules Verne correctly predicted that the aphelion temperature would be nearly that $-60\,°\mathrm{C}$. In the middle — as already with the distances and orbital speeds — he again cut the curve short and did more of a linear interpolation.

Note that in reality, Gallia's temperature will not be the equilibrium one for its current position in general; because the distance from Sun changes continuously and Gallia's matter is "thermally inert" (heat capacity!), the temperature will lag somewhat behind the current solar radiation absorption and won't show that symmetrical pattern anymore. Jules Verne did clearly not consider this phenomenon, and also my calculation assumed stationary temperatures to match up with Verne's statements as well as possible.

References

[1] Densities of Elements. http://www.matpack.de/Info/Nuclear/Elements/density.html, 2010. [Online; accessed 17-April-2010].

[2] Henry Cavendish. Experiments to Determine the Density of the Earth. *Philosophical Transactions of the Royal Society of London*, pages 469–526, 1798.

[3] Jean-Baptiste Joseph Fourier. General Remarks on the Temperature of the Terrestrial Globe and the Planetary Spaces. *American Journal of Science*, 32:1–20, 1837.

[4] Herbert Goldstein. *Klassische Mechanik*. AULA-Verlag Wiesbaden, eigth edition, 1985. German translation by Günter Gliemann.

[5] W. T. Lynn. Cassini and the solar parallax. *The Observatory*, 25:236–238, June 1902.

[6] Isaac Newton. *Philosophiae Naturalis Principia Mathematica*. London, third edition, 1726.

[7] NIST. Saturation (Pressure). http://www.nist.gov/srd/WebGuide/nist10v2.2/NISTIR5078-Tab2.pdf, 2010. [Online; accessed 17-April-2010].

[8] Wilhelm Raith, editor. *Erde und Planeten*, volume 7 of *Bergmann, Schaefer — Lehrbuch der Experimentalphysik*. de Gruyter, second edition, 2001.

[9] Josef Stefan. Über die Beziehung zwischen der Wärmestrahlung und der Temperatur. *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften*, 79:391–428, 1879.

[10] William Thomson. On an Absolute Thermometric Scale founded on Carnot's Theory of the Motive Power of Heat, and calculated from Regnault's Observations. *Philosophical Magazine*, October 1848.

[11] Jules Verne. *From the Earth to the Moon; and, Round the Moon*, volume 83 of *Project Gutenberg*. Project Gutenberg, P.O. Box 2782, Champaign, IL 61825-2782, USA, 1993.

[12] Jules Verne. *Off on a Comet*, volume 1353 of *Project Gutenberg*. Project Gutenberg, P.O. Box 2782, Champaign, IL 61825-2782, USA, 1998.

[13] Wikipédia. Lieue — Wikipédia, l'encyclopédie libre. http://fr.wikipedia.org/w/index.php?title=Lieue&oldid=46418814, 2009. [Online; accessed 26-February-2010].

[14] Wikipedia. Maxwell-boltzmann distribution — Wikipedia, The Free Encyclopedia. http://en.wikipedia.org/w/index.php?title=Maxwell-Boltzmann_distribution&oldid=356338982, 2010. [Online; accessed 17-April-2010].

[15] Wikisource. Hector Servadac — Wikisource. http://fr.wikisource.org/w/index.php?title=Hector_Servadac&oldid=1260106, 2009. [Online; accessed 29-October-2009].